AP Calculus AB

Students must be prepared to take a quiz on pre-calculus material by the 2nd day of class.

You must be able to complete each of these problems **with and without the use of a calculator** (unless otherwise noted) by the start of the school year.

You may contact Mr. Chetlen with any questions at chetlenj@franklinps.net

Section I: Lines

(F-BF 1) Build a function that models a relationship between two quantities

A. Write an equation for the line through the point $P(1, -2)$ with slope $m = 3$.

B. Let line L be $2x + y = 4$. Write an equation for the line through point $P(-1, 4)$ that is (a) parallel to line L and (b) perpendicular to line L.

C. For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$ (a) parallel? (b) perpendicular?

Section II: Functions

(F-LE 4) Construct and compare linear, quadratic, and exponential models and solve problems.

A. State the following for each function. Use interval notation. Sketch a general graph for each function.

a. domain and range
b. interval(s) where the function is increasing/decreasing
c. extrema (maximums/minimums), if any
d. intercepts, if any
e. asymptotes, if any
f. end behavior
g. symmetry (even/odd function), if any

1. quadratic function: $y = ax^2$ for $a < 0$ and $a > 0$
2. cubic function: $y = ax^3$ for $a < 0$ and $a > 0$
3. rational function: $y = \frac{1}{x}$
4. logarithmic function: $y = \ln x$
5. exponential function: $y = e^x$ or $y = e^{-x}$
6. trigonometric functions: $y = \sin x$ $y = \cos x$ $y = \tan x$

(F-IF 7) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

B. Sketch a graph of each function.

7. rational function: $y = \dfrac{3x+4}{x-2}$

8. piecewise function: $y = \begin{cases} x^2 , & x < 0 \\ 1 , & x = 0 \\ x+3 , & x > 0 \end{cases}$

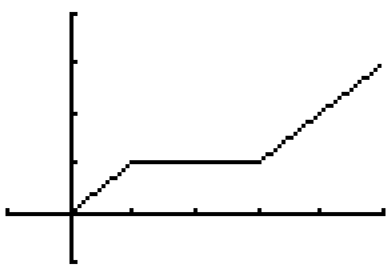
9. absolute value function: $y = |x-2| + 3$

~~10. greatest integer function: $y = \text{int}(x)$ (Math, >, #5)~~

C. Explain whether the function is even, odd, or neither. Show the analysis that leads to your conclusion.

1. $y = x - 3$
2. $y = \sqrt{x^2 + 2}$
3. $y = \dfrac{x^3}{x^2 - 1}$
4. $y = x + \cos x$
5. Show that the product of two even functions is an even function.
6. Show that the product of an even function and an odd function is an odd function.

D. Write a piecewise formula for the function where $0 \le x \le 5$.



E. Construct a piecewise function of $y = 2|x-3|$.

F. Find $f(g(x))$, $g(f(x))$, and $f(f(1))$ if $f(x) = x + 5$ and $g(x) = x^2 - 3$.

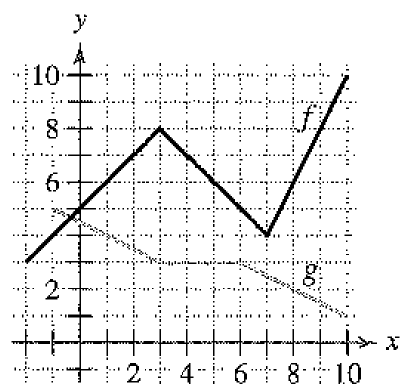
G. Use the table to evaluate each of the following:

x	-6	-4	-2	0	2	4	6
$f(x)$	1	2	3	4	5	6	8
$g(x)$	0	-2	6	-6	2	3	10

1. $f(g(2))$
2. $g(f(-4))$
3. $f(f(4))$
4. $g(f(g(g(0))))$

Is it true that $g(f(x)) = f(g(x))$ for *all* values of x? If false, state a counterexample.

H. Use the graphs of f and g below to evaluate each of the following:



1. $f(3) + g(3)$
2. $fg(3)$
3. $\dfrac{f}{g}(6)$
4. $f(g(6))$
5. $g(f(6))$
6. slope of f at $x = 9$ times $g(4)$
7. $f(f(2))$
8. $g(f(1)) - g^2(3)$

(F-BF. 3) Build new functions from existing functions.
Identify the effect on the graph by replacing *f*(*x*) with *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

I. State each parent function and describe the transformation of the parent function.

1. $y = x^2 + 2x - 2$ 2. $y = 3e^{-x}$ 3. $y = -\sin\left(x + \dfrac{\pi}{4}\right)$ 4. $y = \dfrac{2x+3}{x-1}$

(F-BF 4) Find inverse functions.

J. Find the inverse of each function algebraically.

1. $y = 5x - 1$ 2. $y = \ln x$ 3. $y = \tan 2x$

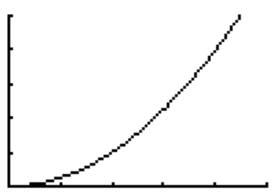
Section III: Slopes

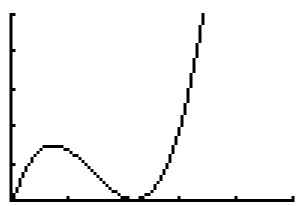
A. Sketch a possible graph of a function with the following slopes and/or asymptotes.

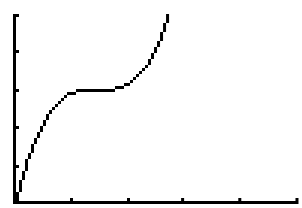
1. The function has a positive slope on the interval $(-\infty,\ 5)$ and a negative slope on the interval $(5,\ \infty)$.
2. The function has a negative slope on the interval $(-\infty,\ -3) \cup (3,\ \infty)$ and a constant slope on the interval $(-3,\ 3)$.
3. The function has a positive slope on the interval $(-\infty,\ 0) \cup (0,\ \infty)$, an undefined slope at $x = 0$, and a horizontal asymptote at $y = 0$.
4. The function has a negative and increasing slope on the interval $(-\infty, 2)$, and a positive and decreasing slope on $(2,\ \infty)$.

B. Match each story to the most appropriate graph. The *x*-axis represents time and the *y*-axis represents distance away from home.

(a) I started on my way to school then realized I forgot my paper. So, I went back to pick it up.
(b) I started on my way to school then realized I was going to be late. So, I sped up.
(c) I started on my way to school then got stopped for speeding. After I received my ticket, I continued on my way to school.

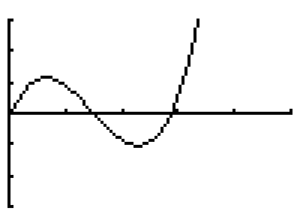
(i)


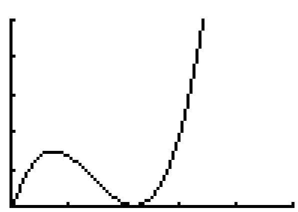
(ii)


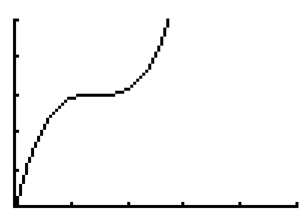
(iii)


C. Match each story to the most appropriate graph. The *x*-axis represents time and the *y*-axis represents velocity.

(a) I started on my way to school then realized I forgot my paper. So, I went back to pick it up.
(b) I started on my way to school then realized I was going to be late. So, I sped up.
(c) I started on my way to school then got stopped for speeding. After I received my ticket, I continued on my way to school.

(i)


(ii)


(iii)


(F-IF 6) Interpret functions that arise in applications in terms of the context.

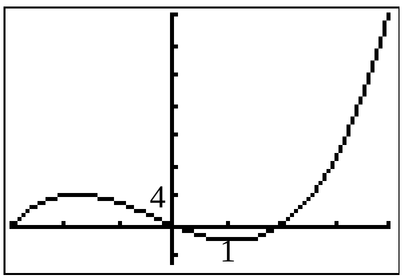
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

D. 1. State the average rate of change between the points (6,-2) and (-1, 12)

2. A cyclist travels 12 miles from home to work. She leaves at 7:02 am and arrives at 7:47. Find her average rate of change (speed) in miles per hour.

3. Find the average rate of change of the function $y = \sqrt{x}$ over the interval [0, 16].

4. Find the average rate of change between the endpoints of the function graphed below.



Section IV: Trigonometry

(F-TF 9) Prove and apply trigonometric identities

A. Know the trigonometric values of the angles of the special right triangles and quadrantal angles.

1. $\sin 0$ 2. $\cos \frac{\pi}{2}$ 3. $\cos \frac{2\pi}{3}$ $\tan \frac{3\pi}{2}$

What is the slope of a line with a $45°$ angle? What is the value of $\tan 45°$? What does the tangent value of an angle have to do with the slope of the line? Why?

Section V: Geometry

Know the formulas and be able to apply for (surface and lateral) area and volume of cubes, cones, cylinders, and spheres.

Section VI: Algebra

(F-LE 4) Construct and compare linear, quadratic, and exponential models and solve problems.

Simplify the following expressions.

1. Show $\dfrac{1}{\dfrac{1-e^x}{1+e^x}}\left(\dfrac{(1+e^x)(-e^x)-(1-e^x)e^x}{(1+e^x)^2}\right)$ simplifies to $\dfrac{-2e^x}{1-e^{2x}}$

2. Show $\dfrac{\sin^2 x}{1+\cos x}$ simplifies to $1-\cos x$

3. Show $\dfrac{x^2(3x^2-2)-(x^3-2x)2x}{\left(x^2\right)^2}$ simplifies to $1+\dfrac{2}{x^2}$

4. Find the *exact value* of x by solving algebraically: $2\ln(x-1)+4=3$

5. Find the *exact value* of x by solving algebraically: $3e^{(x-4)}-1=5$

6. Solve for y: $\ln y-\ln(y-1)=\ln x$

7. Simplify $\ln e$, $\ln e^3$, $\ln 1$, $\log_w w^{45}$

8. Simplify $3\log_3 3-\frac{3}{4}\log_3 81-\frac{1}{3}\log_3 \frac{1}{27}$

Section VII: Application Problems (Calculators may be used.)

(G-GMD 2) Explain volume formulas and use them to solve problems.

A. *Geometry* A cannery will package tomato juice in a 2-liter (2000 cm^3) cylindrical can. Find the radius and height if the can is to have a surface area that is less than 1000 cm^2.

B. *Optimization* A gym wants to build a rectangular swimming pool with the top of the pool having a surface area of 1000 ft^2. The pool is required to have a walk of uniform width 2 ft surrounding it. Let x be the length of one side of the swimming pool.
   a. Express as a function of x the area of the plot of land needed for the pool and surrounding sidewalk.
   b. Find the dimensions of the plot of land that has the least area. What is the least area?

C. *Projectile motion* A little league team has a throwing machine that propels a baseball upward from ground level with an initial velocity of 55 ft/sec.
   a. Does the ball reach a height of 52 ft?
   b. How many seconds after the ball is propelled is it 42 ft above the ground?

D. *Trigonometry* From the top of the 100-ft-tall building a man observes a car moving toward the building. If the angle of depression of the car changes from 22° and 46° during the period of observation, how far does the car travel?