AP Calculus AB - SOLUTIONS

Section I: Lines

A. Use point – slope form, much easier: $y-1=1(x-1)$

B. Slope of *L* is - 2.

(a) $y-2=-2(x+2)$ $y - 4 = -2(x + 1)$

(b) $y-2=\frac{1}{2}(x+2)$ $y - 4 = 1/2\ (x + 1)$

C. Slope of line 1 is $\frac{-2}{k}$. Slope of line 2 is -1.

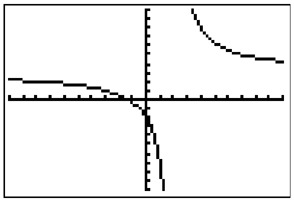
a. $\frac{-2}{k}=-1$, $k = 2$

b. $\frac{-2}{k}=1$, $k = -2$

Section II: Functions

B.

1.



Important Pieces:

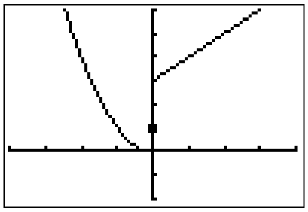
Horizontal Asymptote at $y = 3$

Vertical Asymptote at $x = 2$

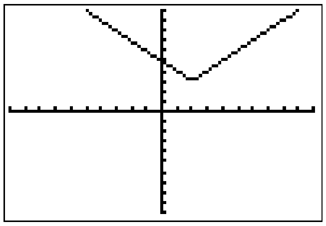
x-intercept at $x=-\frac{4}{3}$

y-intercept at $y=-2$

2.



3.



C.

1. $y = x - 3$; neither

$$f(-x) = (-x) - 3$$
$$= -x - 3$$
$$f(-x) \neq f(x) \ nor \ -f(x)$$

2. $y = \sqrt{x^2 + 2}$; even

$$f(-x) = \sqrt{x^2 + 2}$$
$$= \sqrt{(-x)^2 + 2}$$
$$= \sqrt{x^2 + 2}$$
$$f(-x) = f(x)$$

3. $y = \dfrac{x^3}{x^2 - 1}$; odd

$$f(-x) = \frac{(-x)^3}{(-x)^2 - 1}$$
$$= \frac{-x^3}{x^2 - 1}$$
$$= -\frac{x^3}{x^2 - 1}$$
$$f(-x) = -f(x)$$

4. $y = x + \cos x$; neither

$$f(-x) = (-x) + \cos(-x)$$
$$= -x + \cos x$$
$$f(-x) \neq f(x) \ nor \ -f(x)$$

5. Show that the product of two even functions is an even function.

Let f and g be even functions, then
$$f(x)g(x) = f(-x)g(-x)$$
$$= f(x)g(x)$$
Therefore $f(x)g(x)$ is even.

Proofs such as these will be done together not necessarily on assessments. You should be able to make sense of these, however.

6. Show that the product of an even function and an odd function is an odd function.

Let *f* be an even function and *g* be an odd function, then

$$\begin{aligned} f(x)g(x) &= f(-x)g(-x) \\ &= f(x)\cdot[-g(x)] \\ &= -f(x)g(x) \end{aligned}$$

Therefore $f(x)g(x)$ is odd.

D. $f(x)=\begin{cases} x & 0\le x<1 \\ 1 & 1\le x<3 \\ x-2 & 3\le x\le 5 \end{cases}$

Break this into several segments; from [0,1) then [1,3), then [3, 5).

E. The break is at $x = 3$ because of what's inside the absolute value bars. When $x\le 3$, take the opposite sign of the function. When $x>3$, keep the function as it is with no absolute bars.

$$f(x)=\begin{cases} -2x+6 & if \quad -\infty<x\le 3 \\ 2x-6 & if \quad 3<x<\infty \end{cases}$$

F. $f(g(x))=x^2+2$ $\quad g(f(x))=x^2+10x+22$ $\quad f(f(1))=11$

G.

1. 5; $g(2)=2,\ then\ f(2)=5$
2. 2; $f(-4)=2\ then\ g(2)=2$
3. 8; $f(4)=6\ then\ f(6)=8$
4. 3; $g(0)=-6$, $g(-6)=0,$ $f(0)=4,$ $g(4)=3$

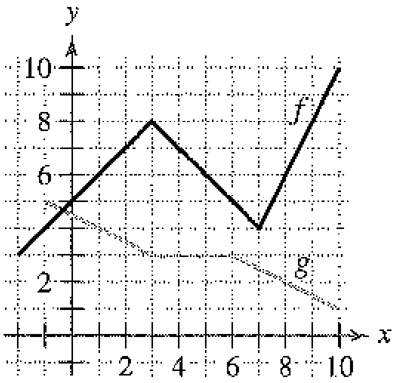
Is it true that $g(f(x))=f(g(x))$ for *all* values of *x*? If false, state a counterexample.

FALSE, $f(g(0))=0\ \ but\ \ g(f(0))=10$

H.

1. $f(3)+g(3)=8+3=11$
2. $fg(3)=8\cdot 3=24$
3. $\dfrac{f}{g}(6)=\dfrac{5}{3}$
4. $f(g(6));\quad g(6)=3\ \ then\ \ f(3)=8$
5. $g(f(6));\quad f(6)=5,\ \ then\ g(5)=3$
6. slope of f at x = 9 times $g(4)$

   slope is 2, times $g(4)$ = 6



7. $f(f(2));\quad f(2)=7,\ then\ \ f(7)=4$

8. -6; $g(f(1))-g^2(3);\qquad f(1)=6\quad g(6)=3,\ \ g(3)=3$

   $3- 3^2 = -6$

I.

1. Complete the square

$$y=x^2+2x+1\ \ -2-1$$

$$y=(x+1)^2-3$$

Parent: $y=x^2$
Transformations: Left 1, Down 3

2. **Parent:** $y=e^x$
   Transformations: Vertical Stretch factor 3, Reflect across the x-axis.

3. **Parent:** $y=\sin x$
   Transformations: Left $\pi/4$, Reflect across the y-axis.

4. First, Long Division:

U

$$y=\frac{2x+3}{x-1}$$

$$\begin{array}{r} 2 \\ x-1\overline{)\,2x+3} \\ -(2x-2) \\ \hline 5 \end{array}$$

$$y=2+\frac{5}{x-1}$$

Parent: $y=\dfrac{1}{x}$

Transformations: Right 1, Vertical stretch factor 5, up 2

Vertical stretch *must* precede up 2. Otherwise the horizontal asymptote would be at 10 and not 2.

J.

Switch *x* and *y*, then solve for *y*.

1. $y = \frac{x+1}{5}$

2. $x = \ln y$

$e^x = y$

3. $x = \tan 2y$

$\tan^{-1} x = 2y$

$\frac{\tan^{-1} x}{2} = y$

Section III: Slopes

A. Sketch a possible graph of a function with the following slopes and/or asymptotes.

1-4 varied.

B. (a) ii	(b) i	(c) iii
C. (a) i	(b) iii	(c) ii

D. This section is all about slope, aka, average rate of change, rise over run, change in *y* over change in *x*, $\frac{y_2 - y_1}{x_2 - x_1}$, *m*. So get two points and calculate.

1. - 2
2. 16 mph
3. $\frac{1}{4}$
4. 4

Section IV: Trigonometry

1. 0
2. 0
3. $-\frac{1}{2}$
4. Undefined

Slope of a $45°$ angled line is 1. $\tan 45° = 1$. They are the same because $\tan\theta = \frac{y}{x}$ which equals $\frac{y_2 - y_1}{x_2 - x_1}$ when the line intersects the origin (0, 0).

Section VI: Algebra

1. Show $\frac{1}{\frac{1-e^x}{1+e^x}}\left(\frac{(1+e^x)(-e^x)-(1-e^x)e^x}{(1+e^x)^2}\right)$ simplifies to $\frac{-2e^x}{1-e^{2x}}$

$$\frac{1+e^x}{1-e^x}\left(\frac{(1+e^x)(-e^x)-(1-e^x)e^x}{(1+e^x)^2}\right)$$

Simplify the outside complex fraction.

$$\frac{\cancel{1+e^x}}{1-e^x}\left(\frac{(1+e^x)(-e^x)-(1-e^x)e^x}{(1+e^x)^{\cancel{2}}}\right)$$

$$\frac{1}{1-e^x}\left(e^x\,\frac{(1+e^x)(-1)-(1-e^x)}{1+e^x}\right)$$

factor out the e^x

$$\frac{1}{1-e^x}\left(e^x\,\frac{-1-e^x-1+e^x}{1+e^x}\right)$$

$$\frac{1}{1-e^x}\left(e^x\,\frac{-2}{1+e^x}\right)$$

$$\frac{-2e^x}{(1-e^x)(1+e^x)}$$

Foil

$$\frac{-2e^x}{1-e^{2x}}$$

2. Show $\dfrac{\sin^2 x}{1+\cos x}$ simplifies to $1-\cos x$

$\dfrac{1-\cos^2 x}{1+\cos x}$ (trig ID)

$\dfrac{(1+\cos x)(1-\cos x)}{1+\cos x}$ (factor)

$1-\cos x$ (cancel)

3. $\dfrac{x^2(3x^2-2)-(x^3-2x)2x}{\left(x^2\right)^2}$

$$\frac{3x^4-2x^2-2x^4+4x^2}{x^4}$$

$$\frac{x^4+2x^2}{x^4}$$

$$\frac{x^4}{x^4}+\frac{2x^2}{x^4}$$

$$1+\frac{2}{x^2}$$

4. $2\ln(x-1)+4=3$

$$2\ln(x-1)=-1$$

$$\ln(x-1)=-\frac{1}{2}$$

$$e^{-1/2}=x-1$$

$$e^{-1/2}+1=x$$

5. $3e^{x-4}-1=5$

$$3e^{x-4}=6$$

$$e^{x-4}=2$$

$$x-4=\ln 2$$

$$x=\ln 2+4$$

6. $\ln y - \ln(y-1) = \ln x$

$$\ln \frac{y}{y-1} = \ln x$$

$$\frac{y}{y-1} = x$$

$$y = x(y-1)$$

$$y = xy - x$$

$$y - xy = -x$$

$$y(1-x) = -x$$

$$y = \frac{-x}{1-x} \quad or \quad \frac{x}{x-1}$$

7. 1; 3; 0; 45
8. 1

Section VII: Application Problems (Calculators may be used.)

A. $\pi r^2 h = 2000$ $\qquad$ $2\pi r^2 + 2\pi r h \le 1000$

$h = \dfrac{2000}{\pi r^2}$ $\qquad$ Graph $2\pi r^2 + 2\pi r \dfrac{2000}{\pi r^2} \le 1000$

$4.619cm < r < 9.655cm$
$6.829cm < h < 29.839cm$

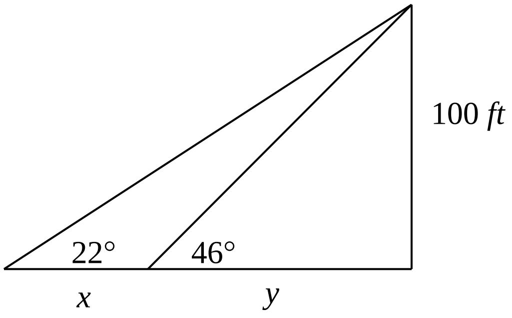
B. Graph $A = (x+4)\left(\dfrac{1000}{x} + 4\right)$ for x: $\qquad$ 31.623 *ft* x 31.623 *ft*

minimum area is approximately 1268.982 ft^2

C. $h(t) = -16t^2 + 55t$

a. no, the ball reaches a height of 47.266 *ft*
b. at x = 1.145 seconds and at x = 2.292 seconds

D. $\tan 22° = \dfrac{100}{x+y}$ $\qquad$ $\tan 46° = \dfrac{100}{y}$



Solve for y and then substitute

$$\tan 22^\circ = \frac{100}{\left(\dfrac{100}{\tan 46^\circ} + x\right)}$$

x = 150.940 *ft*

I graphed each side of the equation and found the intersection. Might be a little quicker.